THE PROBLEM OF THE SCREW CONVEYER FOR FLUIDS OF VARYING VISCOSITY IN THE PRESENCE OF HEAT TRANSFER

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Screw conveyers and presses find wide application in industry. In several papers dealing with the theory of these devices [1,2] the considerable energy dissipation and the effect of temperature on fluid viscosity had not been taken into consideration.

An attempt is made in this article to allow for these effects in a simplified flow model, generally accepted as the first approximation. This model consists of a straight rectangular channel with one sliding wall and a fluid in rotary motion in which there is no pressure gradient along the screw axis, i.e., a screw conveyer free of load is considered.

We assume that the dependence of viscosity on temperature is subject to either the exponential [3] or hyperbolic laws

$$\mu = \mu_0 \exp (U / RT),\ \mu = \mu_0 [1 + \alpha (T - T_0)]^{-1}, \tag{1}$$

where μ_0, U, α, and T_0 are constants, R is the gas constant, and T is the absolute temperature.

We locate the coordinate origin at the left corner of the rectangular channel section of dimensions a and b ($b \le a$), and we express all dimensions in terms of b. The velocity is given in terms of the channel upper (sliding) wall velocity v_0 along the z-axis. Considering the effects of dissipation, but neglecting heat transfer along the channel axis because fluid velocities in screw conveyers are low, we write the equations of motion and energy conservation in the case of the exponential law (1) in dimensionless form as

$$\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\exp\frac{-\theta}{1+\beta\theta}\right)+\frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y}\exp\frac{-\theta}{1+\beta\theta}\right)=0, \tag{2}$$

$$\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\delta\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]\exp\frac{-\theta}{1+\beta\theta}=0,$$

$$w=\frac{v}{v_0},\quad \theta=\frac{U}{RT_0^2}(T-T_0),\quad \beta=\frac{RT_0}{U},$$

$$\delta=\frac{\mu_0 v_0^2 U}{\lambda R T_0^2}\exp\frac{U}{RT_0}. \tag{3}$$

Where w and θ are, respectively, the dimensionless velocity and the temperature; β and δ are dimensionless parameters; λ is the thermal conductivity coefficient.

The boundary conditions for the velocity are: w = 1 at y = 1, and w = 0 at x = 0, x = a/b, and y = 0.

Various conditions can be imposed on the temperature depending on the circumstances, namely:

(1) The temperature of the screw hub and that of the channel are equal, $T = T_0$; then everywhere along the boundary $\theta = 0$.

(2) The screw hub temperature is $T = T_0$, while that of the channel is $T = T_1$; then $\theta = 0$ at y = 1, and $\theta = \theta_1$ along the remaining part of the boundary.

(3) The screw hub temperature is $T = T_0$ and the channel is thermally insulated, i.e., $\partial T/\partial n = 0$; then at y = 1 we have $\theta = 0$ and $\partial\theta/\partial n = 0$ along the remaining part of the boundary.

We substitute in Eq. (2) as follows:

$$\frac{\partial w}{\partial x}=\frac{\partial u}{\partial x}\exp\frac{\theta}{1+\beta\theta},\quad \frac{\partial w}{\partial y}=\frac{\partial u}{\partial y}\exp\frac{\theta}{1+\beta\theta}. \tag{4}$$

Hence, the function u will satisfy the Laplace equation

$$\Delta u = 0. \tag{5}$$

From the integrability of system (4) with respect to w it follows that

$$\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial x}=\frac{\partial\theta}{\partial x}\frac{\partial u}{\partial y}, \tag{6}$$

but then [4]

$$\theta = \Psi(u). \tag{7}$$

Substituting (4) and (7) into (3), and using (5), we obtain for Ψ the ordinary differential equation

$$\frac{d^2\Psi}{d\omega^2}+\delta\exp\frac{\Psi}{1+\beta\Psi}=0. \tag{8}$$

If the viscosity is strongly affected by temperature, $\beta \ll 1$ [5], and we can write

$$\frac{d^2\Psi}{du^2}+\delta\exp\Psi=0. \tag{9}$$

The solution of Eq. (9) is [6]

$$\Psi=\ln[c_1^2\,\mathrm{ch}^{-2} c_1\sqrt{1/2\,\delta}\,(u-c_2)], \tag{10}$$

where c_1 and c_2 are integration constants, and $c_1 > 0$.

From (4) and (7) it follows that

$$w=\int e^{\Psi}du+c. \tag{11}$$

Substituting (10), we obtain

$$w=c_1\sqrt{2/\delta}\,\mathrm{th}\,c_1\sqrt{1/2\,\delta}\,(u-c_2)+c. \tag{12}$$

Obviously u must be also subject to the following boundary conditions: $u = u_1$ for w = 0, and $u = u_0$ for w = 1, where u_1 and u_0 are unknown constants.

A direct check shows that u_1 does not appear in the final expressions for the velocity and temperature fields, hence we immediately assume $u_1 = 0$. We thus have four arbitrary constants needed for satisfying conditions along the boundary. With conditions (1) for the temperature $\theta = \Psi$ satisfied, we obtain the follow-in system of four equations:

$$c_1=\mathrm{ch}\,c_1\sqrt{1/2\,\delta}\,(u_0-c_2),\quad c_1=\mathrm{ch}\,c_1 c_2\sqrt{1/2\,\delta},$$

$$c=c_1\sqrt{2/\delta}\,\mathrm{th}\,c_1\sqrt{1/2\,\delta}\,c_2,$$

$$1-c=c_1\sqrt{2/\delta}\,\mathrm{th}\,c_1\sqrt{1/2\,\delta}\,(u_0-c_2). \tag{13}$$

Solving this system for c, c_1, c_2, and u_0, we obtain

$$c=1/2,\quad c_1=\sqrt{1+1/8\,\delta},\quad c_2=1/2\,u_0, \tag{14}$$

$$u_0=\frac{8}{\sqrt{(8+\delta)\delta}}\ln\left[\sqrt{1+1/8\,\delta}+\sqrt{1/8\,\delta}\right]. \tag{15}$$

The cases of boundary conditions (2) and (3) are dealt with in a similar manner.

Finally we have for the velocity and temperature fields the following dimensionless expressions:

$$\theta=\ln(1+1/8\,\delta)\,\mathrm{ch}^{-2}\,1/4\sqrt{(8+\delta)\delta}\,(u-1/2\,u_0), \tag{16}$$

$$w=1/2+1/2\sqrt{1+1/8\,\delta}\,\mathrm{th}\sqrt{(8+\delta)\delta}\,(u-1/2\,u_0)\,1/4, \tag{17}$$

where u_0 is defined by (15), and u is the solution for the boundary-value problem of the Laplace equation with boundary conditions u =

$= u_0$ at $y = 1$, and $u = 0$ at $x = 0$, $x = a/b$, and $y = 0$, written in the form

$$u = \frac{4u_0}{\pi} \sum_{n=0}^{\infty} \frac{\operatorname{sh}(2n+1)\pi yb/a}{\operatorname{sh}(2n+1)\pi b/a} \frac{\sin(2n+1)\pi bx/a}{2n+1}. \tag{18}$$

Once the velocity field (17) is known, the fluid flow rate can be determined by numerical integration.

We now write the expressions defining the velocity and temperature fields in the case of hyperbolic dependence of viscosity on temperature (1), such as with boundary conditions (2)

$$\theta = (1+\theta_1)\cos\sqrt{\delta}\,u + \frac{\delta - \theta_1(2+\theta_1)}{2\sqrt{\delta}} \sin\sqrt{\delta}\,u - 1, \tag{19}$$

$$w = \frac{1+\theta_1}{\sqrt{\delta}} \sin\sqrt{\delta}\,u + \frac{\delta - \theta_1(2+\theta_1)}{2\delta}(1 - \cos\sqrt{\delta}\,u), \tag{20}$$

in which u is defined by Eq. (18), but u_0 is different and given by

$$u_0 = \frac{1}{\sqrt{\delta}} \operatorname{Arccos} \frac{(2+\theta_1)^2 - \delta}{(2+\theta_1)^2 + \delta}. \tag{21}$$

Here the dimensionless parameter $\delta = \mu_0 v_0^2 \alpha/\lambda$.

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